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UNIT I

Physical Foundations of Biological Systems

Foundational Concept: Complex living organisms transport materials, sense their environment, process signals, and respond to changes using processes understood in terms of physical principles.

CHAPTER 1 Translational Motion, Forces, Work, Energy, and Equilibrium in Living Systems

CHAPTER 2 Importance of Fluids for the Circulation of Blood, Gas Movement, and Gas Exchange

CHAPTER 3 Electrochemistry and Electrical Circuits and Their Elements

CHAPTER 4 How Light and Sound Interact with Matter

CHAPTER 5 Atoms, Nuclear Decay, Electronic Structure, and Atomic Chemical Behavior

Unit I MINITEST

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CHAPTER 1

Translational Motion, Forces, Work, Energy, and Equilibrium in Living Systems

Read This Chapter to Learn About

- Translational Motion
- Forces and Equilibrium
- Work, Energy, and Power

On a scale as small as an atom or as large as a planet, motion is an important constant critical to all living things. It is easy to imagine that if all motion stopped, from the atom to the planets and everything in between, then life would cease to exist. The science of motion is referred to as **kinematics**. This chapter focuses on kinematics and the concepts and equations that describe the motion of objects.

TRANSLATIONAL MOTION

Translational motion describes the motion of an object that moves from one position to another without reference to a fixed point. This explanation of translational motion becomes clear when compared to **rotational motion**, which is the motion of an object that moves from one position to another with reference to a fixed point—an axis. Translational motion includes the motion of an object in a straight line (e.g., a car moving down the road or a ball thrown up in the air), and the curved, parabolic trajectory of a launched or thrown projectile. Kinematics, sometimes referred to as **translational**

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kinematics, represents the set of equations that relate the important physical variables of motion.

Units and Dimensions

In physics, typical quantities that might be measured include the length of a lab table, the mass of a textbook, or the time required for an object to strike the ground when dropped from a known height. These quantities are described by **dimensions** or physical descriptions of a quantity. In physics, three basic dimensions are used, corresponding to the three examples of quantities noted previously: length (L), mass (M), and time (T). These are not the only basic dimensions in physics; in fact, in physics there are seven basic dimensions: **length, mass, time, temperature, amount of a substance, electric current**, and **luminous intensity**.

A quantity may also be described by some combination of these three dimensions. For example, consider the quantity of **force**, defined by Newton's Second Law of Motion as the product of mass and acceleration. The dimension of mass is M, and acceleration (defined as the change in velocity over the change in time) is L/T^2. So the dimension of force is $(M)(L/T^2)$, or ML/T^2.

UNITS OF MEASUREMENT

Although the dimension indicates the type of physical quantity expressed by a physical measurement, units indicate the amount of the physical quantity. Each of the dimensions described in the previous section (i.e., length, mass, and time) is measured in terms of a **unit**, which indicates the amount of a physical quantity. An appropriate unit for a specific quantity depends on the dimension of the quantity. For example, let's say you want to know the length of a pencil. Because the dimension of interest is length, the pencil can be measured in terms of various units, such as centimeters, meters, inches, or feet—all of which describe length. The choice of unit used to describe the length of the pencil depends on the size of the pencil; you probably would not measure the length of a pencil in terms of miles or kilometers.

Although there are several systems of units known in science, the system of units that has been adopted by the science community and that will be used throughout this review is the SI system of units (Système International d'Unités), also known as the **metric system**. In the **SI system of units**, the base unit of length is the **meter**, the base unit of mass is the **kilogram**, and the base unit of time is the **second**, as shown in the following table.

TABLE 1-1 Physical Quantities and SI Units of Measurement

Physical Quantity	Dimension	SI Base Unit
Length	L	meter, m
Mass	M	kilogram, kg
Time	T	second, s

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UNIT CONVERSIONS

All physics equations must be equal in both dimensions and units. For example, consider the quantity speed. The units for speed must be consistent in terms of dimension $\left(\frac{L}{T}\right)$ and in units $\left(\frac{\text{km}}{\text{h}} \text{ or } \frac{\text{m}}{\text{s}}\right)$ in order for the final result to be physically logical. **Conversion factors** convert units through ratios of equivalent quantities from one system of measure to another. Following are some examples of the process of unit conversions:

1. Determine the number of grams (g) in 5 kilograms (kg).

   $$5\text{ kg} \times \frac{1000\text{ g}}{1\text{ kg}} = 5000\text{ g}$$

   In this problem, 5 kg is multiplied by the conversion factor $\frac{1000\text{ g}}{1\text{ kg}}$. The conversion factor is equal to 1 because 1 kg = 1000 g, and any value (e.g., 5 kg) multiplied by 1 retains its value, even if expressed in different units. Although multiplication of a quantity by a conversion factor changes the numeric value of the quantity, it does not change the measurement of the physical quantity, because objects of mass 5 kg and 5000 g are identical. The arrangement of the conversion factor is important, because the final result must have units of grams. In order for this to happen, the conversion factor must have the kilogram unit on the bottom so that the unit cancels with the kilogram unit in 5 kg, yielding a quantity with a unit in grams.
2. Determine the number of seconds in 1 year. To find the number of seconds in 1 year, begin with the quantity of 1 year and multiply that quantity by appropriate conversion factors such that the last unit standing is seconds.

   $$1\text{ year} \times \frac{365\text{ days}}{1\text{ year}} \times \frac{24\text{ hours}}{1\text{ day}} \times \frac{60\text{ minutes}}{1\text{ hour}} \times \frac{60\text{ seconds}}{1\text{ minute}}$$

   $$= \left(\frac{1 \times 365 \times 24 \times 60 \times 60}{1 \times 1 \times 1 \times 1}\right)\text{ seconds} = 3.15 \times 10^7\text{ s}$$

ESTIMATING QUANTITIES USING UNIT CONVERSIONS

You may use this same conversion technique to estimate quantities. However, in making such estimations, you need to make several assumptions—and the more sound and robust your assumptions are, the more realistic your estimations will be.

> **EXAMPLE:** Calculate the number of heartbeats that occur in an individual over a lifetime.
>
> **SOLUTION:** Before you begin to solve this problem, you must make several assumptions including: (1) the average heartbeat rate of the individual over the course of a lifetime = 1.1 beats/s, and (2) the number of years in a lifetime = 80 years. The problem requires you to convert from 1.1 beats per second to the

total number of beats occurring over 80 years.

$$\frac{\text{Number of beats}}{\text{Lifetime (80 years)}} = \frac{1.1 \text{ beats}}{1 \text{ second}} \times \frac{60 \text{ seconds}}{1 \text{ minute}} \times \frac{60 \text{ minutes}}{1 \text{ hour}} \times \frac{24 \text{ hours}}{1 \text{ day}} \times \frac{365 \text{ days}}{1 \text{ year}} \times \frac{80 \text{ years}}{1 \text{ lifetime}}$$

$$= \left(\frac{1.1 \times 60 \times 60 \times 24 \times 365 \times 80}{1 \times 1 \times 1 \times 1 \times 1 \times 1}\right) \frac{\text{beats}}{\text{lifetime}}$$

$$= 2.78 \times 10^9 \frac{\text{beats}}{\text{lifetime}}$$

By this estimate, the number of heartbeats in a person's lifetime of 80 years is 2.78 billion beats. However, this calculation is an estimate of a quantity, and not an exact answer, primarily because of factors that were not addressed, such as: (1) the extra days not factored in during leap years; (2) the change in heart rate that occurs between night and day; (3) the effect of different emotions, medications, and illnesses; (4) family history, genetic disorders, and illnesses that affect the circulatory system; and (5) the moment that the heart begins to beat includes the time during fetal development in which the defined circulatory system begins circulating blood within the fetus (typically in the second trimester).

Vectors and Scalars

In physics, measurements of physical quantities, processes, and interactions can be classified according to two types. Some measurements, such as displacement, velocity, and acceleration, require both a magnitude (size) as well as a direction, while some measurements, such as distance and speed, are presented only as a magnitude or size. Those quantities that require both a magnitude and direction are known as **vector quantities**, whereas those quantities that require only magnitude are referred to as **scalar quantities**. Examples of scalar and vector quantities follow.

Scalar Quantities	Vector Quantities
Measurement	
Time	
Mass	
Area	
Volume	
Kinematics	
Distance	Displacement
Speed	Velocity
	Acceleration

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Scalar Quantities	Vector Quantities
Dynamics	
Work	Force
Energy	Momentum
	Torque

A vector quantity is generally noted by a boldfaced letter, sometimes with an arrow drawn over the top of the letter. For example, a vector quantity S can be represented by the symbols, **S** or $\vec{\mathbf{S}}$. The magnitude of the vector quantity S is typically indicated by the vector quantity symbol encased within the absolute value bars, |**S**| or $|\vec{\mathbf{S}}|$. In addition, a vector quantity is represented graphically by an arrow, with the size of the arrow corresponding to the size or magnitude of the vector quantity and the orientation of the arrow corresponding to the direction of the vector quantity, as shown in Figure 1-1.

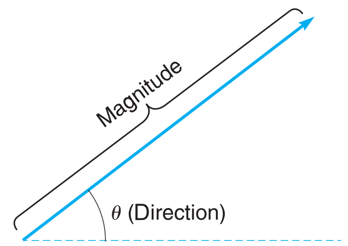

FIGURE 1-1 Representation of a vector quantity.

Vector Addition and Subtraction

Although scalar and vector quantities are similar in that they both can be added or subtracted, they are different in that scalar quantities can be added or subtracted arithmetically, but vector quantities must be added or subtracted in ways that take into account their direction as well as their magnitude. One method of vector addition is the **head-to-tail method**. A second method is called the **component method**.

HEAD-TO-TAIL METHOD OF VECTOR ADDITION

Let's say you have two vectors, **A** and **B**. In this method of vector addition, one of the vectors, **A**, is drawn to scale with its tail positioned at the origin. At the head of this vector, the tail of the second vector is drawn according to its scale. The **vector sum** or **resultant**, indicated by **R**, is the magnitude and direction of the arrow drawn from the tail of the first vector to the head of the second vector. This method can be extended to include more than two vectors.

If the two vectors **A** and **B** are one-dimensional (i.e., lie along the same dimension), then they may be added as shown in Figure 1-2.

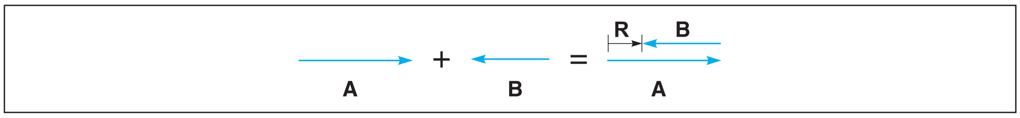

FIGURE 1-2 Adding one-dimensional vectors.

If the vectors are perpendicular to one another (i.e., vector **A** lies along the x-axis and vector **B** lies along the y-axis, as shown in Figure 1-3), then the resultant vector, **R**, represents the hypotenuse of the right triangle formed by the two perpendicular vectors, **A** and **B**. The magnitude of **R** can be found using the Pythagorean theorem:

$$c^2 = a^2 + b^2 \quad \text{or} \quad R^2 = A^2 + B^2$$

The following trigonometric function is used to determine the direction of the resultant vector:

$$\theta = \tan^{-1}\left(\frac{\text{opp}}{\text{adj}}\right) = \tan^{-1}\left(\frac{B}{A}\right)$$

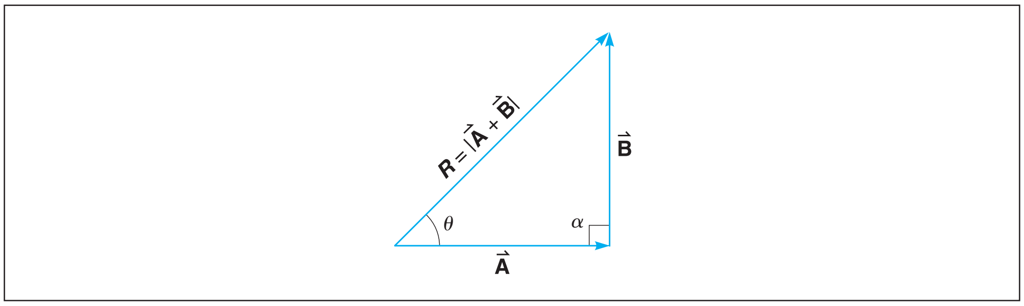

FIGURE 1-3 Adding perpendicular vectors. *Source:* From George Hademenos, *Schaum's Outline of Physics for Pre-Med, Biology, and Allied Health Students*, McGraw-Hill, 1998; reproduced with permission of The McGraw-Hill Companies.

COMPONENT METHOD OF VECTOR ADDITION

The component method of vector addition requires that the x and y components of each vector be determined. In a two-dimensional coordinate system, a vector can be positioned solely along the x direction, solely along the y direction, or can have components both in the x and y directions. A vector quantity oriented at an angle implies that a component of the quantity is in the x direction and a component of the quantity is in the y direction, as shown in Figure 1-4. To determine exactly how much of a vector is in the x or the y directions, you use the following trigonometric functions to resolve the vector (**A**) in terms of its horizontal or x components (A_x) and its vertical or y components (A_y):

Horizontal component: $\cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{A_x}{A}$

Vertical component: $\sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{A_y}{A}$

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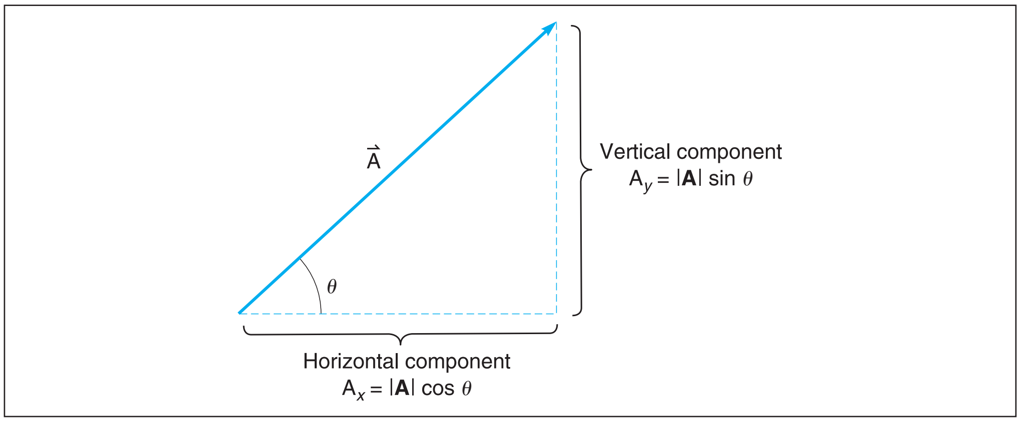

FIGURE 1-4 Component method of vector addition.

The angles $\theta = 90°$, $180°$, $270°$, and $360°$ are special cases. Assuming that $\theta = 0°$ corresponds to the $+x$ axis,

- A vector **A** at 90° has the components: $A_x = 0$; $A_y = +|\mathbf{A}|$
- A vector **A** at 180° has the components: $A_x = -|\mathbf{A}|$; $A_y = 0$
- A vector **A** at 270° has the components: $A_x = 0$; $A_y = -|\mathbf{A}|$
- A vector **A** at 360° has the components: $A_x = +|\mathbf{A}|$; $A_y = 0$

Nevertheless, each component of the vectors is added, with the sum of the x components now representing the x component of the resultant vector, R_x, and the sum of the y components representing the y component of the resultant vector, R_y. Once you know the x and y components of the resultant vector, you can determine the magnitude of the resultant vector by using the Pythagorean theorem:

$$R^2 = R_x^2 + R_y^2 \quad \text{or} \quad R = \sqrt{R_x^2 + R_y^2}$$

You can determine the direction by using the trigonometric function:

$$\theta = \tan^{-1}\left(\frac{\text{opp}}{\text{adj}}\right) = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

EXAMPLE: On a walk across a large parking lot, a shopper walks 25.0 m to the east and then turns and walks 40.0 m to the north. Find the resultant of the two displacement vectors representing the shopper's walk using: (1) the head-to-tail method of vector addition and (2) the component method of vector addition.

SOLUTION: The first leg of the shopper's walk, which we call vector **A**, is **A** = 25.0 m east. The second leg, which we call vector **B**, is **B** = 40.0 m north.

1. Using the head-to-tail method of vector addition, the two vectors are drawn as shown in Figure 1-5. You can calculate the magnitude of the resultant vector **R** drawn from the tail of vector **A** to the head of vector **B** by using the

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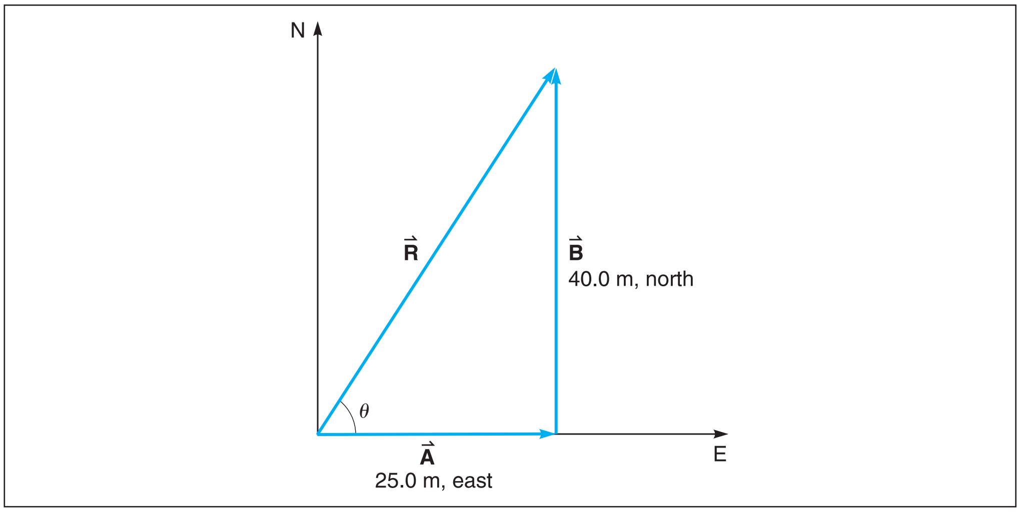

FIGURE 1-5 Diagram for shopper's walk.

Pythagorean theorem:

$$R^2 = A^2 + B^2 = (25.0\text{ m})^2 + (40.0\text{ m})^2 = 2225\text{ m}^2$$

$$R = \sqrt{2225\text{ m}^2} = 47.2\text{ m}$$

You can determine the direction or angle of the resultant vector **R** from the trigonometric relationship:

$$\theta = \tan^{-1}\left(\frac{\text{opp}}{\text{adj}}\right) = \tan^{-1}\left(\frac{B}{A}\right) = \tan^{-1}\left(\frac{40.0\text{ m}}{25.0\text{ m}}\right)$$

$$= 57.9^\circ \text{ (as measured from the positive } x\text{-axis)}$$

2. Use the component method of vector addition to resolve vectors **A** and **B** into their x and y components:

Vector **A**:	$A_x = 25.0\text{ m}$	$A_y = 0\text{ m}$
Vector **B**:	$B_x = 0\text{ m}$	$B_y = 40.0\text{ m}$
Vector **R**:	$R_x = A_x + B_x$	$R_y = A_y + B_y$
	$= 25.0\text{ m} + 0\text{ m}$	$= 0\text{ m} + 40.0\text{ m}$
	$= 25.0\text{ m}$	$= 40.0\text{ m}$

You can find the magnitude of the resultant vector **R** by using the Pythagorean theorem:

$$R^2 = R_x^2 + R_y^2 = (25.0\text{ m})^2 + (40.0\text{ m})^2 = 2225\text{ m}^2$$

$$R = \sqrt{2225\text{ m}^2} = 47.2\text{ m}$$

The direction or angle of the resultant vector **R** can be determined from the trigonometric relationship:

$$\theta = \tan^{-1}\left(\frac{\text{opp}}{\text{adj}}\right) = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{40.0\text{ m}}{25.0\text{ m}}\right)$$

$$= 57.9^\circ \text{ (as measured from the positive } x\text{-axis)}$$

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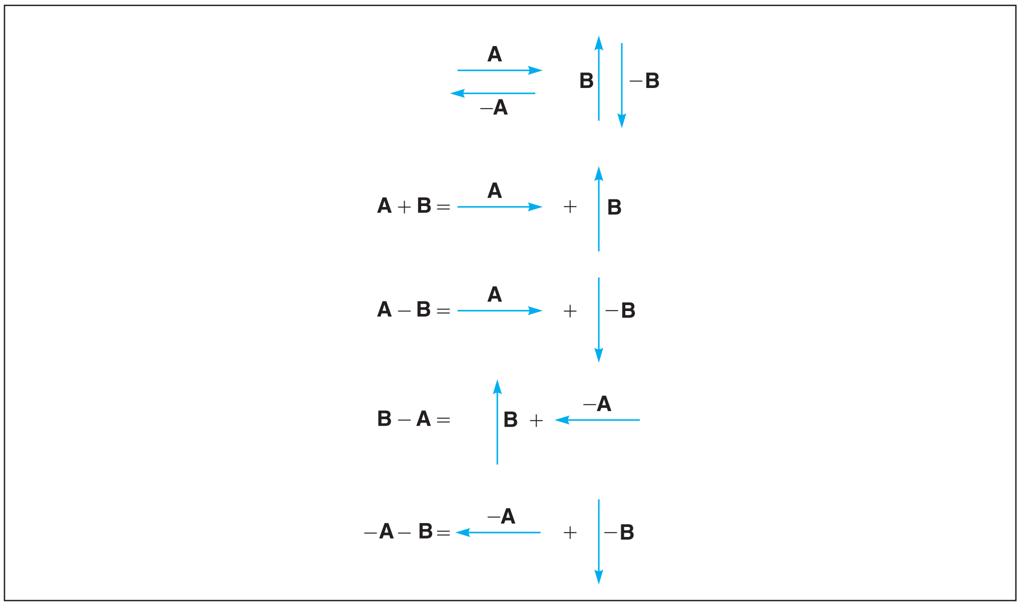

FIGURE 1-6 Vector subtraction.

VECTOR SUBTRACTION

Subtraction of a vector can be done by reversing the direction of the vector and adding it to the remaining vectors, as shown in Figure 1-6. Reversing the sign of the vector simply changes the direction of the vector without affecting its magnitude.

Displacement, Velocity, and Acceleration

Displacement (Δx) is defined as a distance in a given direction. Consider a typical trip to work in which you travel from point A (home, starting position) to point B (work, final position). Expressed in units of length (e.g., meters, kilometers, yards, or miles), displacement is a measure of the **length** [difference between the final position (x_f) and the initial position (x_i)] required to get from point A to point B, regardless of the path taken.

$$\text{Displacement: } \Delta x = x_f - x_i$$

Because displacement is a directed distance, it is possible that the displacement can be either positive or negative. A **negative displacement**, just as is the case for any negative quantity in physics, does not imply that the distance is a negative quantity; rather, it implies that the direction of displacement is opposite to that direction considered positive.

A similar quantity related to length is distance. Although displacement is the length between the final point and starting point, **distance** is the length required to get from the starting point to the ending point, dependent on the path taken. If you leave home, drop by the post office to mail some letters, take your children to school, and stop

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by the grocery store before reaching work, the path taken is much longer than the straight path between home and work. In this case, the distance (home → post office → school → grocery store → work) is much larger than the displacement (home → work).

Speed

Speed (s) is the rate of change of distance over a time interval and expressed in dimensions of length per unit time.

Velocity (v) is the rate of change of displacement over a time interval or the rate at which a directed distance between point A and point B is covered over an interval of time. Velocity, also expressed in dimensions of length per unit time, is the speed of an object in a given direction.

$$\text{Average velocity: } v = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

Average velocity represents velocity over a time interval. **Instantaneous velocity** represents velocity at a given instant of time, similar to the velocity of a car indicated by its speedometer.

Acceleration (a) is the rate at which velocity changes. Acceleration, expressed in dimensions of length per unit time squared, is the change of velocity in a given direction over a defined time interval.

$$\text{Average acceleration: } a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

Average acceleration represents average velocity over a time interval. **Instantaneous acceleration** represents acceleration at a given instant of time.

Graphical Representation of Motion

In a typical motion graph, time [typically in seconds (s)] is always represented along the x-axis (horizontal), whereas the variable along the y-axis could be displacement [in meters (m), for example] or velocity [in meters per second (m/s), for example]. If there is an accompanying table with data points that are represented in a graph, the left column contains values representative of the **independent variable** (typically time) which are plotted along the horizontal or x-axis, whereas the right column contains values of the **dependent variable** (either displacement or velocity, respectively) which are plotted along the vertical or y-axis.

If you graph displacement versus time for an object, the velocity of the object is the slope of the line on the graph:

$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{displacement (meters)}}{\text{time (seconds)}} = \text{velocity}\left(\frac{\text{meters}}{\text{second}}\right)$$

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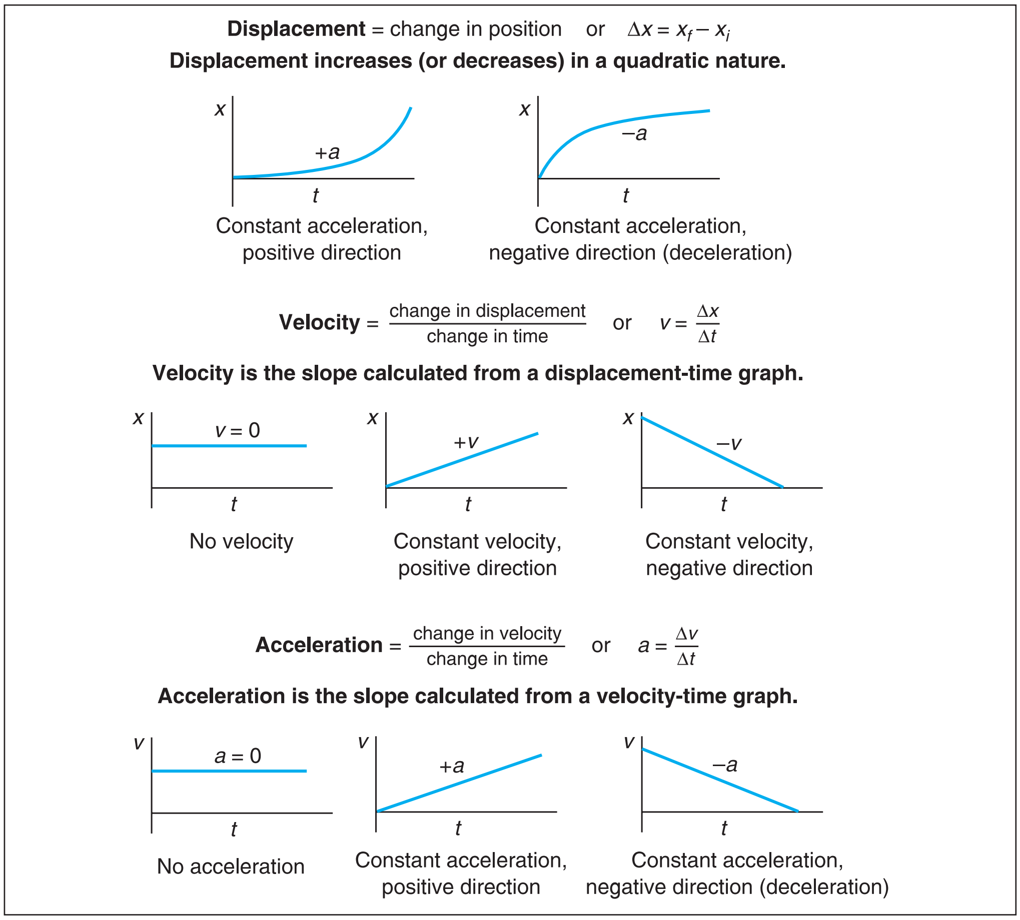

FIGURE 1-7 Graphs of displacement, velocity, and acceleration.

If you graph velocity versus time for an object, the acceleration of the object is represented by the slope of the line on the graph:

$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{velocity (meters/second)}}{\text{time (seconds)}}$$
$$= \text{acceleration}\left(\frac{\text{meters}}{\text{second}^2}\right)$$

Velocity and acceleration represent a change in a variable (displacement or velocity, respectively) as a function of time. Figure 1-7 shows examples of graphs of displacement, velocity, and acceleration. Note that the slope and the y-intercept of each graphed quantity depend on the location of the object and the rate of change of motion of the object.

As you evaluate and interpret the graphs of motion data, you can obtain important information by considering the following questions:

- What is the physical significance of the data?
  - What do the graphed data tell me about the nature and behavior of an object?
  - What are the quantities presented in the columns of data found in a table and/or the quantities represented along the x- and y-axes of a graph?

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- If the x-axis is time and the y-axis is displacement (distance in a given direction), then the slope of the graph gives velocity.
- If the x-axis is time and the y-axis is velocity (speed in a given direction), then the slope of the graph gives acceleration.

➤ Does y change as x is varied? If so, how?

- If there is a straight horizontal displacement—time graph, there is no change in y, and thus the velocity equals zero.
- If the graph is linear, the slope is constant; therefore, the velocity is constant.
- If the graph is nonlinear (increasing/decreasing in a quadratic nature), the slope is changing, indicating either a positive acceleration or a negative acceleration (deceleration).

➤ If there is motion, what is its quantitative value (in magnitude and units)?

- The quantitative value of motion implies the numeric value of the slope of the graph.
- If the x-axis is time and the y-axis is displacement, then the slope of the graph yields velocity, as calculated next.

$$\begin{aligned}\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} &= \frac{\text{displacement (meters)}}{\text{time (seconds)}} \\ &= \text{velocity}\left(\frac{\text{meters}}{\text{second}}\right)\end{aligned}$$

- If the x-axis is time and the y-axis is velocity, then the slope of the graph yields acceleration, as calculated next.

$$\begin{aligned}\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} &= \frac{\text{velocity (meters/second)}}{\text{time (seconds)}} \\ &= \text{acceleration}\left(\frac{\text{meters}}{\text{second}^2}\right)\end{aligned}$$

Uniformly Accelerated Motion

Up to this point, we have defined and discussed motion in terms of displacement, velocity, and acceleration. In order for you to be able to use these quantities and apply them to physics problems, you must be able to determine relations between them. For motion of an object with constant uniform acceleration along the x-axis, you can apply the following four equations:

1. Displacement with constant uniform acceleration:

$$\Delta x = \frac{1}{2}\left(v_i + v_f\right)\Delta t$$

$$\text{Displacement} = \frac{1}{2}\,(\text{initial velocity} + \text{final velocity})\,(\text{time interval})$$

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2. Final velocity with constant uniform acceleration:

$$v_f = v_i + a\Delta t$$

$$\text{Final velocity} = \text{initial velocity} + (\text{acceleration})\,(\text{time interval})$$

3. Displacement with constant uniform acceleration:

$$\Delta x = v_i \Delta t + \frac{1}{2} a(\Delta t)^2$$

$$\text{Displacement} = (\text{initial velocity})\,(\text{time interval}) + \frac{1}{2}\,(\text{acceleration})\,(\text{time interval})^2$$

4. Final velocity after any displacement:

$$v_f^2 = v_i^2 + 2a\Delta x$$

$$(\text{Final velocity})^2 = (\text{initial velocity})^2 + 2(\text{acceleration})\,(\text{displacement})$$

You might ask why there are two equations used to calculate final velocity and two equations used to calculate displacement. Careful inspection of the equations indicates that each equation depends on four different variables. This gives you maximum flexibility to determine any related quantity of motion about an object given a specific scenario, depending on the given information.

EXAMPLE: A car initially moving at 20 m/s uniformly accelerates at 2.5 m/s^2. Find the final speed and displacement of the car after 8.0 s.

SOLUTION: The given information from this problem includes:

$$v_i = 20 \text{ m/s} \quad a = 2.5 \text{ m/s}^2 \quad \Delta t = 8.0 \text{ s}$$

To determine the final speed of the car, use the following equation:

$$v_f = v_i + a\Delta t = 20\,\frac{\text{m}}{\text{s}} + \left(2.5\,\frac{\text{m}}{\text{s}^2}\right)(8.0 \text{ s}) = 40\,\frac{\text{m}}{\text{s}}$$

To determine the displacement of the car, use the following equation:

$$\Delta x = v_i \Delta t + \frac{1}{2} a(\Delta t)^2 = \left(20\,\frac{\text{m}}{\text{s}}\right)(8.0 \text{ s}) + \frac{1}{2}\left(2.5\,\frac{\text{m}}{\text{s}^2}\right)(8.0 \text{ s})^2 = 240 \text{ m}$$

Free-Fall Motion

The equations for uniformly accelerated motion presented previously dealt with motion primarily moving along the x-axis (from left to right, and vice versa). However, the same concepts, definitions, variables, and equations also apply to motion along the y-axis (from up to down, and vice versa). In the case of motion along the y-axis, a constant acceleration is caused by gravity. Any and all types of motion involving the y-axis, such as throwing a ball in the air or dropping a pencil, occur under the influence of gravity. But what is it about gravity that factors into the motion of such objects? **Gravity** is a force that pulls objects to Earth. That pull causes the objects to accelerate in a constant manner. The acceleration due to gravity, g, is given numerically by

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$g = -9.8\ \text{m/s}^2$ and always acts downward. Thus, the four equations of motion given previously that described motion of an object in the x-direction can also be applied to an object moving in the y-direction by making two minor changes: (1) displacement that was originally noted in the equations of motion as Δx now becomes Δy; and (2) the acceleration, a, now is replaced by g, the acceleration due to gravity.

1. Displacement with constant uniform acceleration:

$$\Delta y = \frac{1}{2}\left(v_i + v_f\right)\Delta t$$

2. Final velocity with constant uniform acceleration:

$$v_f = v_i + g\Delta t$$

3. Displacement with constant uniform acceleration:

$$\Delta y = v_i\Delta t + \frac{1}{2}g(\Delta t)^2$$

4. Final velocity after any displacement:

$$v_f^2 = v_i^2 + 2g\Delta y$$

A special case of motion along the y-axis is **free-fall motion**, which refers to motion of an object that is dropped from a certain height or y-direction and allowed to fall toward the ground. The first equation indicates how far an object has fallen per given time:

$$\Delta y = \frac{1}{2}g(\Delta t)^2$$

$$(\text{Distance in the } y\text{-direction}) = \frac{1}{2}\,(\text{acceleration due to gravity})(\text{time in flight})^2$$

Note that this is the same equation as No. 3, displacement with constant uniform acceleration, with the initial velocity $v_i = 0$.

The other equation indicates how fast an object falls per given time:

$$v_f = g\Delta t$$

$$(\text{Velocity in the } y\text{-direction}) = (\text{acceleration due to gravity})(\text{time in flight})$$

Note that this is the same equation as the second equation, final velocity with uniform acceleration, with the initial velocity $v_i = 0$.

EXAMPLE: A stone is dropped from a bridge that is 15 m in height. Determine the stone's velocity as it strikes the water below.

SOLUTION: The given information from this problem includes:

- $v_i = 0$ m/s, because it is dropped or released from rest
- $a = -9.8\ \text{m/s}^2$, because it is accelerating downward as a result of gravity
- $\Delta y = -15.0$ m (This height is negative, because the origin is the object at the top of the bridge, and as it moves downward, it is moving in the negative direction.)

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To determine the final speed of the stone, use the following equation:

$$v_f^2 = v_i^2 + 2g\Delta y = \left(0\ \frac{\text{m}}{\text{s}}\right)^2 + 2\left(-9.8\ \frac{\text{m}}{\text{s}^2}\right)(-15.0\ \text{m}) = 294\ \frac{\text{m}^2}{\text{s}^2}$$

$$v_f = \sqrt{294\ \frac{\text{m}^2}{\text{s}^2}} = 17.1\ \frac{\text{m}}{\text{s}}$$

Strategies for solving one-dimensional motion problems are described next.

STRATEGIES FOR SOLVING ONE-DIMENSIONAL MOTION PROBLEMS

1. Identify and list all given information (known and unknown variables).
   - Do not assume that all information given in a problem is required to solve the problem.
   - Look for key words that might be just as important as values: **rest**, **drop** implies $v_i = 0$, and **stop** implies $v_f = 0$. Also, words like **constant**, **speed up**, **increase**, **slow down**, and **decrease** are important in describing the acceleration of the object.
   - Problems commonly involve the sign of g. Because gravity acts downward, g should be a negative quantity. However, care should be taken to ensure the proper sign convention of all other quantities as well. For example, an object dropped from a cliff will have fallen −5 m after 1 second. The negative value implies a distance in the $-y$ direction.
2. Make sure all units are consistent (in SI system of units), and if not, perform required conversions.
3. Choose an equation that can be solved with the known variables as noted in the following table.

TABLE 1-2

Involved Variables	Equation of Motion in *x* Direction	Equation of Motion in *y* Direction	Involved Variables
$\Delta x, v_i, v_f, \Delta t$	$\Delta x = \frac{1}{2}(v_i + v_f)\Delta t$	$\Delta y = \frac{1}{2}(v_i + v_f)\Delta t$	$\Delta y, v_i, v_f, \Delta t$
$v_i, v_f, \Delta t, a$	$v_f = v_i + a\Delta t$	$v_f = v_i + g\Delta t$	$v_i, v_f, \Delta t, g$
$\Delta x, v_i, \Delta t, a$	$\Delta x = v_i\Delta t + \frac{1}{2}a(\Delta t)^2$	$\Delta y = v_i\Delta t + \frac{1}{2}g(\Delta t)^2$	$\Delta y, v_i, \Delta t, g$
$v_i, v_f, \Delta x, a$	$v_f^2 = v_i^2 + 2a\Delta x$	$v_f^2 = v_i^2 + 2g\Delta y$	$v_i, v_f, \Delta y, g$

   Each problem should include information about four variables. Match these four variables with the outer columns to find the correct equation.

4. Substitute all variables in proper units in the chosen equation, perform the necessary algebraic operations, arrive at the solution, and then ask yourself: Does the answer make sense?

MCAT-3200184 book November 12, 2015 11:23 MHID: 1-25-958837-8 ISBN: 1-25-958837-2

FORCES AND EQUILIBRIUM

Up to now, this chapter has been discussing kinematics, or the concepts and equations that describe the motion of an object being uniformly accelerated. The variables and equations that describe an object's motion in one dimension (along the x- and y-axes) were defined and applied to specific problems and examples. This section explains the causes and the factors involved in motion.

Forces in Nature

Given any object, whether an electron, a person, or a planet, what causes the object to move? The answer is simple—a force. **Force** can be defined simply as a push or a pull. Forces cause objects to move. Forces occur everywhere in nature on every object—but the mere presence of a force does not mean that an object will necessarily move. It is electric forces that are responsible for the movement of current through an electric circuit, mechanical forces that can cause a person to move, and gravitational forces that cause planets to move. The reasons are discussed later as part of Newton's Laws of Motion.

A force is a vector quantity expressed in a unit called the **newton** (N):

$$1 \text{ newton} = 1\,\frac{\text{kilogram}\cdot\text{meter}}{\text{second}^2} \quad \text{or} \quad 1\,\frac{\text{kg}\cdot\text{m}}{\text{s}^2}$$

All forces, whether they are electric, mechanical, or gravitational, are expressed in units of newtons, regardless of their source. Gravitational forces are discussed later in this chapter and electric forces are discussed in Chapter 3. Weight, normal force, friction, and tension are four specific types of mechanical forces that exist in nature and are defined as follows:

1. **Weight** is the force exerted on an object by gravity. It is also referred to as the **force due to gravity**. Any object that has mass has weight. Weight can be calculated by the equation:

   $$\text{Weight} = (\text{mass})\ (\text{acceleration due to gravity})$$

   $$\boldsymbol{W} = m\boldsymbol{g}$$

2. **Normal force** is the force exerted on an object by a surface. It is also referred to as the **support force**. Normal force always acts perpendicular to the surface that is supporting the object.
3. **Friction** is a force generated by the properties of the interface between a moving object and a surface. Friction acts in a direction opposite to an object's motion. The force due to friction, $\mathbf{F}_f$, is defined as:

   $$\text{Frictional force} = (\text{coefficient of friction})\ (\text{normal force})$$

   $$\mathbf{F}_f = \mu\mathbf{N}$$

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There are two types of frictional forces corresponding to the state of motion of the object. If the object is stationary, then a frictional force (static friction) is acting on the object to prevent motion, described by:

$$\mathbf{F}_{f,s} \leq \mu_s \mathbf{N}$$

where μ_s is the **coefficient of static friction**. If the object is set in motion (i.e., subject to an applied force that is greater in magnitude and opposite in direction than the static frictional force), then the object is subject to kinetic (sliding) friction, defined as:

$$\mathbf{F}_{f,k} = \mu_k \mathbf{N}$$

Static friction is generally greater in magnitude than kinetic friction because an object requires a larger force to start an object in motion than it does to be kept in motion.

4. **Tension** is the force exerted by a string, rope, or cable on a suspended object.

Newton's Three Laws of Motion

Before you can understand Newton's Laws of Motion, you must make a distinction between three physical terms that are often confused: mass, inertia, and weight.

- **Mass**, measured in SI units of kilograms, is the amount of substance that an object has.
- **Inertia** is an object's resistance to motion. What makes an object resistant to motion is not an object's size, but its mass. The more mass an object has, the more inertia it has, making the object more resistant to motion. Inertia does not have a physical unit, but is indicated by the object's mass.
- **Weight**, a force exerted on an object due to gravity, is often confused with mass. These are two very different quantities. Weight is a vector quantity expressed in units of newtons, whereas mass is a scalar quantity expressed in units of kilograms. The weight of an object is calculated by multiplying the mass of the object times the acceleration due to gravity, or

  $$\text{Weight} = (\text{mass})\,(\text{acceleration due to gravity})$$

  The mass of an object does not change unless material is added or taken away. The weight of an object can change, for example, on a different planet, where the acceleration due to gravity is different. The mass of a person on Earth is the same as the mass of that person on Mars, as the mass always remains the same. However, the weight of the person differs because each planet has a different gravitational field and hence a different value of g (the acceleration due to gravity). Because the g on Mars is $g = -3.8\ \text{m/s}^2$, there is less of a pull on the person due to gravity, and hence the weight of the person on Mars is less.

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NEWTON'S FIRST LAW OF MOTION

An object at rest remains at rest, and an object in motion remains in motion unless acted upon by an unbalanced force.

Newton's first law is also known as the **law of inertia**. **Inertia** is an object's resistance to motion. The more inertia an object has, the harder it is to move the object. Thus, the more mass an object has, the more inertia it has, and the harder it is for you to move that object.

EXAMPLE: When you are in a car driving down the highway going 65 mph, not only is the car going 65 mph, but you and everyone else in the car are also moving at a velocity of 65 mph. The car continues to move at a velocity of 65 mph. Now let's say that the driver runs into a telephone pole, bringing the car to a complete stop. The telephone pole stops the car, but what about the driver and passengers? The pole does not stop the driver and passengers, so they continue moving at 65 mph until something does stop them—usually the steering wheel and the windshield.

NEWTON'S SECOND LAW OF MOTION

The acceleration of an object is directly proportional to the net external force acting on the object, and inversely proportional to the object's mass.

In equation form, Newton's second law of motion can be expressed as

$$\Sigma \mathbf{F} = m\boldsymbol{a}$$

$$\text{Force} = (\text{mass})\ (\text{acceleration})$$

Force is measured in units of newtons (N), where $1\ \text{N} = 1\ \text{kg} \cdot \text{m/s}^2$. The Σ is the Greek capital letter sigma and signifies a sum. In this context, the sum of the forces equals an object's mass times its acceleration. If the sum of the forces acting on an object is 0, then the object is balanced and in **equilibrium**. Also, because force is a vector, this equation applies to forces acting along the x and y directions, or

$$\Sigma \text{F}_x = ma_x$$

$$\Sigma \text{F}_y = ma_y$$

EXAMPLE: Golf ball versus bowling ball: Which hits the ground first? When released from rest from the same height at the same time, which object will strike the ground first—the golf ball or the bowling ball? Any object released from rest and dropped is accelerated at a constant rate by gravity ($g = -9.8\ \text{m/s}^2$). Because g is constant, the force due to gravity is smaller for the golf ball because of the smaller mass of the golf ball in comparison to the bowling ball. In other words,

$$g = \frac{F_{\text{golf ball}}}{m_{\text{golf ball}}} = \frac{F_{\text{bowling ball}}}{m_{\text{bowling ball}}} = \text{constant}$$

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Gravity exerts a greater pulling force on the greater mass such that its value remains constant. Therefore, neglecting air resistance, the golf ball and the bowling ball hit the ground at the same time.

NEWTON'S THIRD LAW OF MOTION

For every action force exerted on an object, there is an equal yet opposite reaction force exerted by the object.

This law is also known as the **law of action and reaction**.

EXAMPLE: A person who is standing exerts a force on the ground equal to his or her weight. The ground, in turn, exerts an equal yet opposite force on the person, supporting the weight of the person. The force exerted by the ground on the person is the normal force, as defined previously.

Free-Body Diagrams

An important technique used to solve problems involving forces acting on an object is free-body diagrams. A **free-body diagram** is a diagram in which an object is isolated, and all forces acting on the object are identified and represented on the diagram.

In creating free-body diagrams, you should:

- Isolate the object in an imaginary coordinate system in which the object represents the origin.
- Identify and represent all forces (with appropriate magnitude and direction) acting on the object.
- Resolve all forces in terms of their x and y components.
- Substitute into the appropriate Newton's second law of motion, either $\Sigma F_x = ma_x$ or $\Sigma F_y = ma_y$, setting the sum of all forces in each direction equal to ma if the object is accelerating and equal to 0 if the object is not accelerating.
- Solve for the unknown variable.

An example of using free-body diagrams to solve a problem is in the case where an object of mass m is stationary on a ramp with a coefficient of static friction μ_s and inclined at an angle θ, as shown in Figure 1-8.

The object is at rest on the inclined plane of $\theta = 30°$, and you are asked to determine the coefficient of static friction, μ_s, between the object and the inclined plane. The coordinate system is rotated such that the x-axis is now along the incline. As noted in the free-body diagram, there are three forces identified from the scenario: the weight of the object acting down (for which there is a component acting in the x direction and a component in the y direction), the normal force acting perpendicular to the inclined plane, and the frictional force that acts opposite to the direction of motion.

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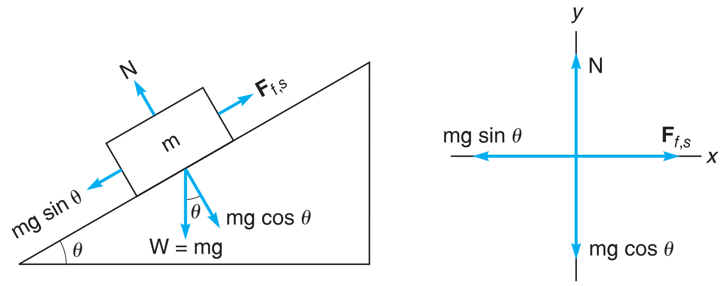


Note: In the free-body diagram, the normal *xy* coordinate system is rotated by an angle θ such that *x* is now parallel to the incline and *y* is now perpendicular to the incline. Therefore, the force of gravity is no longer in the –*y* direction.

FIGURE 1-8 Example of a free-body diagram. *Source:* From George Hademenos, *Schaum's Outline of Physics for Pre-Med, Biology, and Allied Health Students,* McGraw-Hill, 1998; reproduced with permission of The McGraw-Hill Companies.

Substituting these forces into Newton's second law:

$$\Sigma \mathrm{F}_x = ma_x \qquad \Sigma \mathrm{F}_y = ma_y$$

$$\mathbf{F}_{f,s} - mg\sin\theta = 0 \qquad \mathbf{N} - mg\cos\theta = 0$$

By definition, the kinetic frictional force is $\mathbf{F}_{f,s} = \mu_s\mathbf{N}$, where $\mathbf{N}$ can be found from the forces acting in the y direction: $\mathbf{N} = mg\cos\theta$. So the frictional force is:

$$\mathbf{F}_{f,s} = \mu_s\mathbf{N} = \mu_s mg\cos\theta$$

Substituting this expression back into the equation for the forces acting in the x direction:

$$\mu_s\, mg\cos\theta - mg\sin\theta = 0$$

Eliminating mg and solving for μ_s:

$$\mu_s\ \cos\theta - \sin\theta = 0$$

$$\mu_s = \frac{\sin\theta}{\cos\theta} = \tan\theta = \tan 30^\circ = 0.58$$

Examples of several common scenarios are presented in the following table:

TABLE 1-3

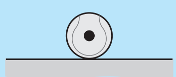
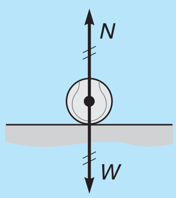
Scenario	Free-Body Diagram	Application of Newton's Second Law
An object in equilibrium lying on a level surface 		Object is in equilibrium, thus $\Sigma\mathbf{F} = m\boldsymbol{a} = 0$ x component: None y component: $\Sigma \mathrm{F}_y = m\,a_y = 0$ $N - W = 0$ $N = W = mg$

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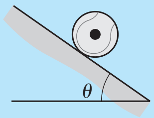
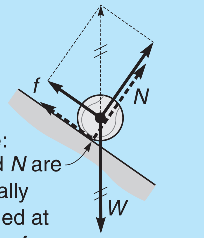
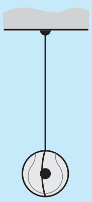
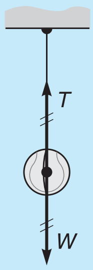
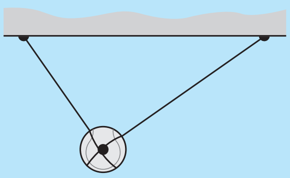
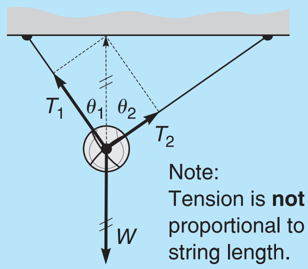
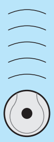
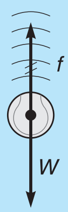
TABLE 1-3 (cont.)

Scenario	Free-Body Diagram	Application of Newton's Second Law
An object in equilibrium lying on an inclined surface with friction preventing motion  	 	Object is in equilibrium, thus $\Sigma \mathbf{F} = m\mathbf{a} = 0$ x component: $\Sigma F_x = ma_x = 0$ $-f + W \sin\theta = 0$ $f = W \sin\theta$ y component: $\Sigma F_y = ma_y = 0$ $N - W\cos\theta = 0$ $N = W\cos\theta$
An object in equilibrium suspended by a string 	 	Object is in equilibrium, thus $\Sigma \mathbf{F} = m\mathbf{a} = 0$ x component: None y component: $\Sigma F_y = ma_y = 0$ $T - W = 0$ $T = W = mg$
An object in equilibrium suspended by two strings of unequal lengths 	 	Object is in equilibrium, thus $\Sigma \mathbf{F} = m\mathbf{a} = 0$ x component: $\Sigma F_x = ma_x = 0$ $-T_1 \sin\theta_1 + T_2 \sin\theta_2 = 0$ $T_1 \sin\theta_1 = T_2 \sin\theta_2$ y component: $\Sigma F_y = ma_y = 0$ $T_1 \cos\theta_1 + T_2 \cos\theta_2 - W = 0$ $T_1 \cos\theta_1 + T_2 \cos\theta_2 = W$
An object is falling and subject to no friction 	 	Object is in motion, thus $\Sigma \mathbf{F} = m\mathbf{a}$ x component: None y component: $\Sigma F_y = ma_y$ $-W(= mg) = ma_y$
An object is falling at constant (terminal) velocity 	 	Object is in equilibrium, thus $\Sigma \mathbf{F} = m\mathbf{a} = 0$ x component: None y component: $\Sigma F_y = ma_y = 0$ $f - W = 0$ $f = W = mg$

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Uniform Circular Motion and Centripetal Force

An object moving with constant speed in a circular path undergoes uniform circular motion. Because the object is moving at constant speed does not mean the object is not accelerating—in fact, it is. Although the magnitude of velocity (speed) is constant for an object in uniform circular motion, it is constantly changing direction. If the object is changing direction, it has a changing velocity and thus is accelerating. This acceleration, called **centripetal** (or center-seeking) **acceleration**, is perpendicular to the tangential velocity and is directed toward the center of rotation. The magnitude of centripetal acceleration, a_c, is defined as:

$$a_c = \frac{v^2}{r}$$

and has typical units of acceleration (m/s^2).

Because an object in uniform circular motion has mass and is accelerating, there is a force directed inward toward the center. It is referred to as the **centripetal force** (F_c) defined by:

$$F_c = ma_c = m\frac{v^2}{r}$$

A centripetal force is a vector quantity and has units of newtons.

Translational and Rotational Equilibrium

An object can be in either translational and/or rotational equilibrium, depending on the forces acting on the object. An object is in **translational equilibrium** when the sum of forces is equal to zero, or

$$\Sigma \mathbf{F} = 0$$

or

$$\Sigma F_x = 0, \qquad \Sigma F_y = 0$$

Another type of force, when exerted on an object, causes rotational motion. This type of force is referred to as **torque**, characterized by the symbol τ. It occurs when an external force acts at a given distance from a fixed pivot point (also known as a lever arm, r), as shown in Figure 1-9. Torque is defined as:

$$\text{Torque} = (\text{force})(\text{lever arm}) \sin\theta$$
$$\boldsymbol{\tau} = \mathbf{Fr} \sin\theta$$

where θ is the angle between the direction of the force and the direction of the lever arm. Because it is a rotational force, torque can also be expressed analogous to the translational force ($\mathbf{F} = m\boldsymbol{a}$) by

$$\boldsymbol{\tau} = I\boldsymbol{\alpha}$$

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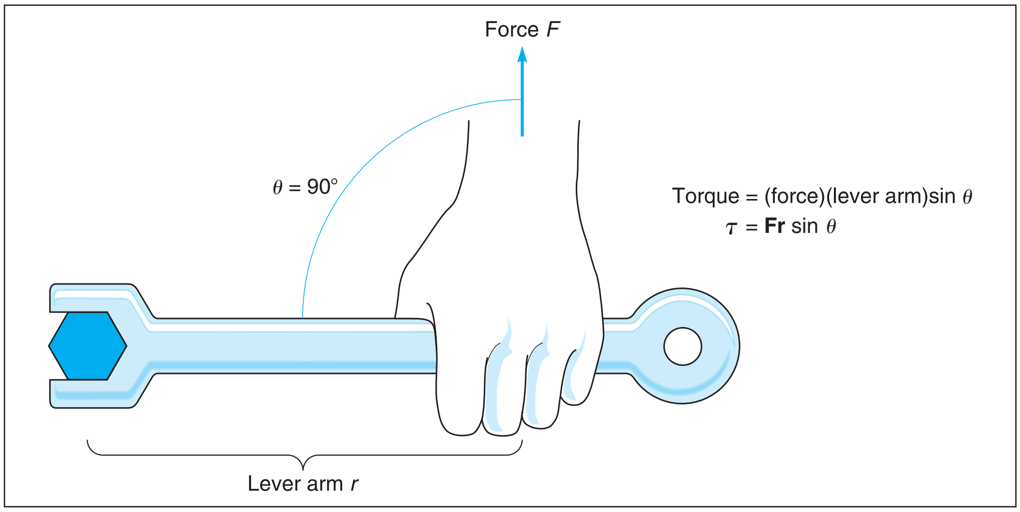

FIGURE 1-9 Torque.

where I is the moment of inertia or an object's resistance to rotational motion and α is the angular acceleration of the rotating object. Torque is a vector quantity with SI units of newton · meter (N · m).

Torque occurs as the result of the rotation of an object, and an object can rotate in either a clockwise or a counterclockwise direction, resulting in τ_{cw} and τ_{ccw}, respectively. To determine the direction of the torque, you must first identify an axis of rotation and the point where each external force acts on the object. Once each torque about the axis of rotation has been calculated, torque is positive if the force causes a counterclockwise rotation; torque is negative if the force causes a clockwise rotation. An object is in **rotational equilibrium** when the sum of torques or rotational forces about any point is equal to zero, or

$$\Sigma \tau = 0$$

or

$$\Sigma \tau_{cw} = \Sigma \tau_{ccw}$$

EXAMPLE: Suppose Tori of mass 42 kg and Cori of mass 70 kg join their sister Lori of mass 35 kg on a seesaw. If Tori and Lori are seated 2.0 and 3.5 m from the pivot, respectively, on the same side of the seesaw, where must Cori sit on the other side of the seesaw in order to balance it?

SOLUTION: The seesaw is in rotational equilibrium, defined by the equation $\Sigma \tau = 0$ or

$$\tau_{\text{Tori}} + \tau_{\text{Lori}} + \tau_{\text{Cori}} = 0$$

or

$$\tau_{\text{Tori}} + \tau_{\text{Lori}} = -\tau_{\text{Cori}}$$

Torque is given as $\tau = \mathbf{Fr} \sin \theta$, where θ is 90° because the girls are exerting a force (equal to their weight) perpendicular to the level arm, as shown in Figure 1-10.

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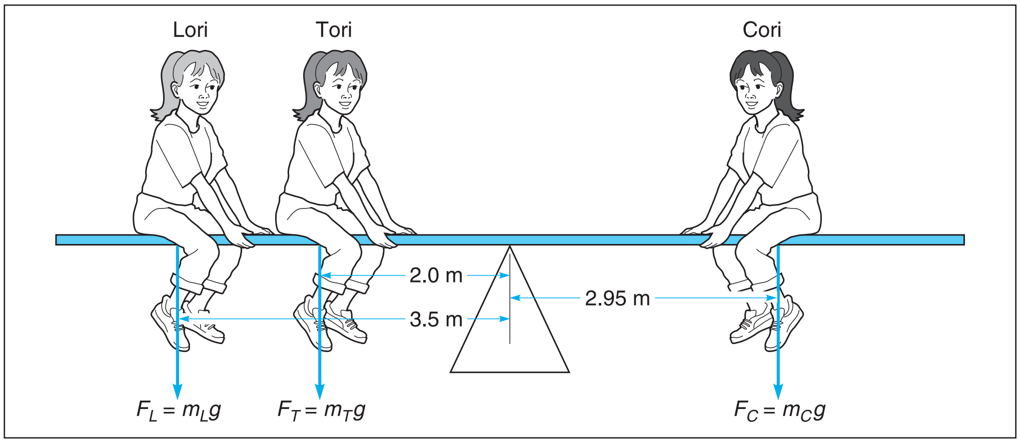

FIGURE 1-10 Seesaw in rotational equilibrium.

Thus,

$$(\mathbf{Fr})_{\text{Tori}} + (\mathbf{Fr})_{\text{Lori}} = -(\mathbf{Fr})_{\text{Cori}}$$

$$(42\text{ kg})(9.8\text{ m/s}^2)(2.0\text{ m}) + (35\text{ kg})(9.8\text{ m/s}^2)(3.5\text{ m})$$

$$= -(70\text{ kg})(9.8\text{ m/s}^2)(\boldsymbol{r}_{\text{Cori}})$$

Solving for $\boldsymbol{r}_{\text{Cori}}$

$$\boldsymbol{r}_{\text{Cori}} = -2.95\text{ m}$$

with the minus sign indicating a position on the opposite side of the seesaw.

Translational and Rotational Motion

Translational motion, or the motion of an object caused by forces exerted in the direction of motion, has already been described through Newton's three laws of motion. These laws of motion can also be extended to describe the **rotational motion** of an object, as noted here:

TABLE 1-4

	Translational Motion	Rotational Motion
Newton's First Law	An object at rest or in motion remains at rest or in motion unless acted upon by an unbalanced force.	A body in rotational motion will continue its rotation until acted upon by an unbalanced torque.
Newton's Second Law	The magnitude of force is equal to the product of an object's mass and its acceleration or $\mathbf{F} = m\boldsymbol{a}$.	The magnitude of a torque is equal to the product of an object's moment of inertia and its angular acceleration or $\tau = I\alpha$.
Newton's Third Law	For every action force acting on an object, there is a reaction force exerted by the object equal in magnitude and opposite in direction to the original force.	For every torque acting on an object, there is another torque exerted by the object equal in magnitude and opposite in direction to the original torque.

MCAT-3200184 book November 12, 2015 11:23 MHID: 1-25-958837-8 ISBN: 1-25-958837-2

WORK, ENERGY, AND POWER

A force applied to an object results in an acceleration of the object in the direction of the force. This is explained by Newton's second law and, in effect, explains why the object is set into motion. How the object responds to this force, ultimately resulting in motion, is the basis for this section.

Work

Work represents the physical effects of an external force applied to an object that results in a net displacement in the direction of the force. Work is a scalar quantity and is expressed in SI units of joules (J) or newton meters (N · m).

By definition, the work done by a constant force acting on a body is equal to the product of the force **F** and the displacement **d** that occurs as a direct result of the force, provided that **F** and **d** are in the same direction. Thus

$$W = Fd$$

$$\text{Work} = (\text{force})(\text{displacement})$$

If **F** and **d** are not parallel but **F** is at some angle θ with respect to **d**, then

$$W = Fd\cos\theta$$

because it is only the portion of the force acting in the direction of motion that causes the object to move its displacement **d**. When the entire force is exerted on an object parallel to its direction of motion, then $\theta = 0^\circ$, and thus $\cos 0^\circ = 1$. This in turn results in the formula $\mathbf{W} = \mathbf{Fd}$ (when **F** is parallel to **d**). When **F** is perpendicular to **d**, $\theta = 90^\circ$ and $\cos 90^\circ = 0$. No work is done in this case.

EXAMPLE: Calculate the work done by a mother who exerts a force of 75 N at an angle of 25° below the horizontal (x-axis) in pushing a baby in a carriage a distance of 20 meters (m).

SOLUTION: The given information in this problem includes:

$$\mathbf{F} = 75\text{ N} \quad \theta = -25^\circ \quad \mathbf{d} = 20\text{ m}$$

Substituting this given information into the equation $W = Fd\cos\theta$ yields

$$W = Fd\cos\theta = (75\text{ N})(20\text{ m})\left[\cos(-25^\circ)\right] = 1360\text{ J}$$

EXAMPLE: Work can also be done in stretching a spring. **Hooke's law** describes the force required to stretch a spring by a displacement x and is represented by the equation:

$$F = -kx$$

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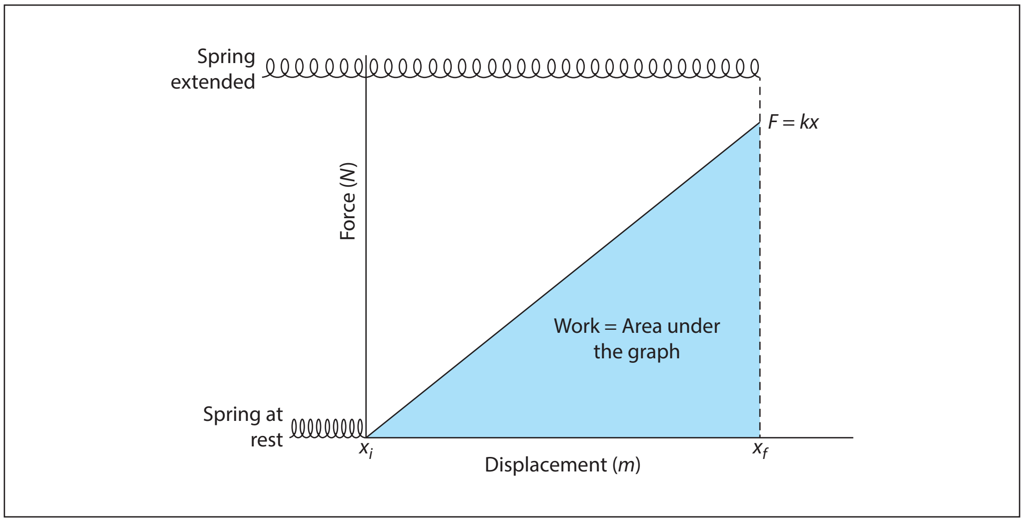

FIGURE 1-11 Work done in stretching a spring.

where k is the spring constant of a spring or the amount of force required to stretch or compress a spring by a defined displacement x. The minus sign corresponds to the opposite direction of the elastic force exerted by a spring in its tendency to return to its normal, unstretched state in response to a stretching force. Since the stretching of a spring requires the application of an external stretching force to pull the spring by a displacement x, work is done in stretching a spring as shown in Figure 1-11.

The work done in stretching a spring can be determined by calculating the area under the graph, which is represented by a triangle of base, x, and height, F. Given that x_i is 0 since the spring position is assumed to be at rest:

$$\text{Area} = \frac{1}{2}(\text{base}) \times (\text{height}) = \frac{1}{2}(x_f) \times (F) = \frac{1}{2}(x_f) \times (kx_f) = \frac{1}{2}kx_f^2 = \frac{1}{2}kx^2$$

$$\text{Area} = \text{work done in stretching spring} = \frac{1}{2}kx^2$$

Pulley Systems

Simple machines are devices that allow people to make work easier by changing the direction of the force applied to the load. One type of simple machine is the **pulley**, which consists of a rope threaded through a groove within the rim of a wheel. As the user applies a force (effort) to one end of a rope threaded over a pulley moving the string a certain distance (effort distance), the other end of the pulley containing the attached load is moved a certain distance (load distance). The pulley reduces the amount of force required to move a load over a given distance. This effect is magnified with the number of pulleys in a pulley system. As the number of pulleys and supporting ropes

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increases for a pulley system, a smaller amount of force is required, although the user must pull the rope over a greater distance.

Two terms that describe the capability of a pulley are its mechanical advantage and its efficiency. The **mechanical advantage**, MA, of a pulley is the amount by which the force required to move a load is reduced. It is described by:

$$\mathrm{MA} = \frac{F_{\mathrm{out}}}{F_{\mathrm{in}}}$$

where F_{out} is the output force (force exerted by the pulley system on the load) and F_{in} is the input force (force exerted on the pulley system by the person/machine).

The **efficiency** of a pulley is the ratio comparing the magnitude of the work output, W_{out}, relative to the work input, W_{in}, or

$$\text{Efficiency} = \frac{W_{\mathrm{out}}}{W_{\mathrm{in}}} = \frac{F_{\mathrm{out}}\, d_{\mathrm{out}}}{F_{\mathrm{in}}\, d_{\mathrm{in}}}$$

EXAMPLE: Two movers who are using a pulley system to lift a dining table of mass 230 kilograms (kg) by a distance of 3.5 m exert a force of 750 N. If the pulley system has an efficiency of 72%, determine the length of rope that must be pulled to accomplish the move.

SOLUTION: The equation needed to solve this problem is:

$$\text{Efficiency} = \frac{W_{\mathrm{out}}}{W_{\mathrm{in}}} = \frac{F_{\mathrm{out}}\, d_{\mathrm{out}}}{F_{\mathrm{in}}\, d_{\mathrm{in}}}$$

where d_{in} represents the unknown quantity. The force exerted by the pulley system on the dining table is equal to the weight of the dining room table, or

$$F_{\mathrm{out}} = \text{weight of the table} = m_{\mathrm{table}}\, g$$

Therefore,

$$d_{\mathrm{in}} = \frac{F_{\mathrm{out}}\, d_{\mathrm{out}}}{F_{\mathrm{in}}\,(\text{efficiency})} = \frac{(230\ \mathrm{kg})\left(9.81\ \dfrac{\mathrm{m}}{\mathrm{s}^2}\right)(3.5\ \mathrm{m})}{(750\ \mathrm{N})\,(0.72)} = 14.6\ \mathrm{m}$$

Work Done by Gases

The physical concept of work can also be applied to gases. Consider Figure 1-12, where a piston is moving in a sealed, gas-filled cylindrical chamber.

As heat is applied to the chamber, the thermal energy of the heat source is transferred to the kinetic energy of the atoms of the gas. This causes expansion of the volume of the gas, which in turn does work on the piston, causing it to be displaced by Δy. The result is work done by a gas on a system (in this case, a piston). Using the following

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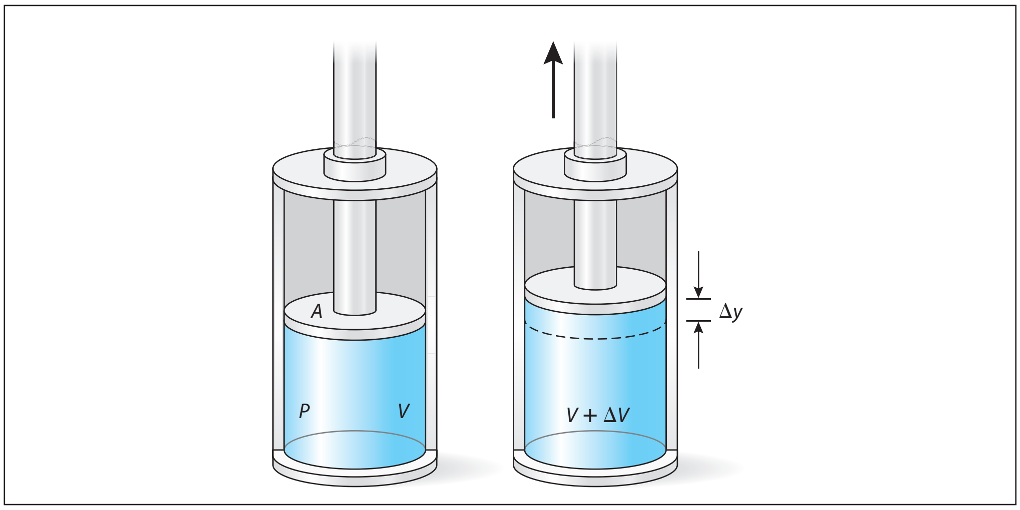

FIGURE 1-12 Piston moving in a sealed, gas-filled cylindrical chamber.

equations:

$$\text{work} = \text{force} \times \text{displacement} \qquad W = F \times d = F \times \Delta y$$

$$\text{pressure} = \frac{\text{force}}{\text{area}} \qquad P = \frac{F}{A} \quad \text{or} \quad F = P \times A$$

then,

$$W = F \times \Delta y = (P \times A) \times \Delta y = P \times (A \times \Delta y) = P\Delta V$$

The work done by the gas on the system is the area under the $P\Delta V$ curve. For the work done by the gas on the piston noted previously, the $P\Delta V$ graph and the calculated work would look like Figure 1-13.

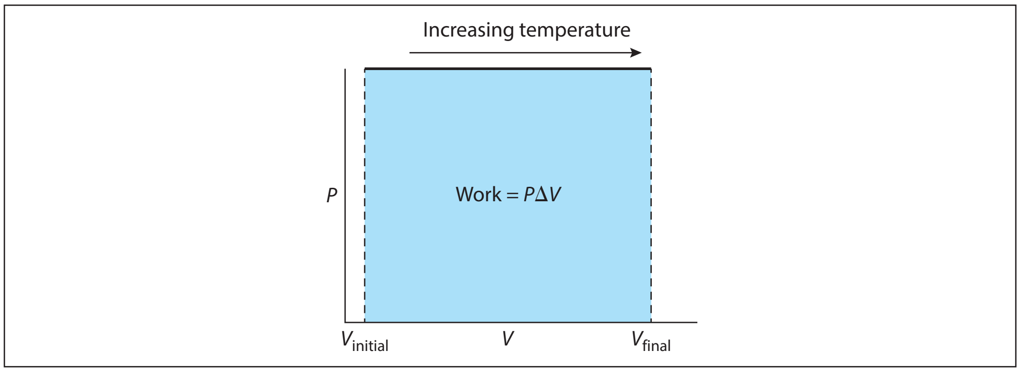

FIGURE 1-13 Work done by the gas on the piston.

Mechanical Energy: Kinetic and Potential

Work describes the effect of a force exerted on an object, causing the object to move a given displacement. Work is performed on a continual basis by athletes when they throw, hit, or kick objects; by children when they slide down a slide, swing on a swing,

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or jump off a jungle gym; and by cars when they drive down the road. What is it that allows athletes, children, or cars to perform work? The answer is energy. Energy exists in many different forms, such as mechanical energy, solar energy (energy provided by the sun), chemical energy (energy provided by chemical reactions), elastic potential energy (energy provided by elastic objects such as a spring or rubber band), thermal energy (energy provided as a result of a temperature difference), sound energy (energy transmitted by propagating sound waves), radiant energy (energy transmitted by light waves), electric energy, magnetic energy, atomic energy, and nuclear energy.

Energy is a property that enables an object to do work. In fact, **energy** is defined as the ability of an object to do work. The more energy an object has, the more work the object can perform. The unit of measure for energy is the same as that for work, namely the joule. The next form of energy we discuss is mechanical energy, which exists in two types: kinetic energy and potential energy.

Kinetic energy is the energy of a body that is in motion. If the body's mass is m and its velocity is v, its kinetic energy is:

$$\text{Kinetic energy (KE)} = \frac{1}{2}mv^2$$

EXAMPLE: Determine the kinetic energy of a 1500-kg car that is moving with a velocity of 30 m/s.

SOLUTION:

$$\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}(1500\text{ kg})(30\text{ m/s})^2 = 6.75 \times 10^5\text{ J}$$

The energy that is stored within a body because of its location above the Earth's surface (or reference level) is called **gravitational potential energy**. The gravitational potential energy of a body of mass m at a height h above a given reference level is:

$$\text{Gravitational potential energy (GPE)} = mgh$$

where g is the acceleration due to gravity. Gravitational potential energy depends on the object's mass and location (height above a level surface), but does not depend on how it reaches this height. In other words, gravitational potential energy is path independent. Any force, such as the force due to gravity, that performs work on an object independent of the object's path of motion is termed a **conservative force**.

EXAMPLE: A 4.5-kg book is held 80 centimeters (cm) above a table whose top is 1.2 m above the floor. Determine the gravitational potential energy of the book (1) with respect to the table and (2) with respect to the floor.

SOLUTION:

1. Here, $h = 80\text{ cm} = 0.8\text{ m}$, so

$$\text{GPE} = mgh = (4.5\text{ kg})(9.8\text{ m/s}^2)(0.8\text{ m}) = 35.3\text{ J}$$

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2. The book is $h = 0.8 \text{ m} + 1.2 \text{ m} = 2.0 \text{ m}$ above the floor, so its GPE with respect to the floor is:

$$\text{GPE} = mgh = (4.5 \text{ kg})(9.8 \text{ m/s}^2)(2.0 \text{ m}) = 88.2 \text{ J}$$

Work–Kinetic Energy Theorem

Work and energy are interrelated quantities. Energy is the ability to do work, and work can only be done if energy is present. A moving object has kinetic energy and maintains the same kinetic energy if it proceeds at the same speed. However, if the object speeds up or slows down, work is required. The net work required to change the kinetic energy of an object is given by the **work–kinetic energy theorem**:

Net work = change in kinetic energy

$$W_{\text{net}} = \Delta\text{KE}$$

Conservative Versus Nonconservative Forces

The forces mentioned earlier in this chapter are all similar in that they represent a push or a pull and can be expressed in units of Newtons. However, they differ in one respect—forces can be conservative or nonconservative. A **conservative force** describes a force that does work on an object independent of its path of motion between its initial and final positions. An example of a conservative force is the gravitational force. If one lifts a book from the floor (Position A) to a height corresponding to a point directly above the floor (Position B), negative work is being done against the force of gravity. At that point (Position B) above the floor, the work done in raising the book was transferred into gravitational potential energy. The book would have the same amount of gravitational potential energy at Position B, regardless of its path to get to Position B. If the book is then released from Position B, gravity does positive work as the book's gravitational potential energy is transferred into kinetic energy as it falls to the ground. Other examples of conservative forces include the elastic force exerted by a spring and the electric force.

A **nonconservative force** describes a force that does work on an object dependent on its path of motion between its initial and final positions. An example of a nonconservative force is friction. Consider, for example, a book that is pushed across a floor. Friction acts on the book, performing negative work from its initial pushing point (Position A) to the final resting point (Position B). If a greater push is exerted on the book, the book moves a greater distance and hence friction acts on the book for a greater amount of time. In other words, the work done by friction on the moving book depends on the path of the book. Other examples of nonconservative forces include tension, normal force, and air resistance.

MCAT-3200184 book November 12, 2015 11:23 MHID: 1-25-958837-8 ISBN: 1-25-958837-2

Conservation of Energy

Energy is a conserved quantity. According to the **law of conservation of energy**, energy can neither be created nor destroyed, only transformed from one kind to another. Examples of this law can be found in many instances in nature, including the physiology of the human body and the working principles of a car. Consider a child on a swing. At the highest point of the arc, the child's energy is exclusively gravitational potential energy, because the child is not moving. As the child then begins moving downward toward the bottom of the arc, the gravitational potential energy of the child is transformed into kinetic energy. At the bottom point of the arc, the child's energy is exclusively kinetic energy.

In general, the initial mechanical energy (sum of potential energy and kinetic energy before an interaction) is equal to the final mechanical energy (sum of potential energy and kinetic energy after an interaction), or

$$\text{Initial mechanical energy} = \text{final mechanical energy}$$

$$(\text{PE}_{\text{initial}} + \text{KE}_{\text{initial}}) = (\text{PE}_{\text{final}} + \text{KE}_{\text{final}})$$

EXAMPLE: A child of mass 25.0 kg is at the top of a slide that is 3.8 m high. What is the child's velocity at the bottom of the slide? (Ignore the effects of friction).

SOLUTION: The child's velocity v at the bottom of the slide can be determined from the law of conservation of energy. The gravitational potential energy of the child at the top of the slide is transformed into kinetic energy at the bottom of the slide.

$$\text{GPE}_{\text{initial}} = \text{KE}_{\text{final}}$$

or, simplifying,

$$mgh = \frac{1}{2}mv^2$$

Solving for v

$$v = \sqrt{2gh} = \sqrt{(2)\left(9.8\ \text{m/s}^2\right)(3.8\ \text{m})} = \sqrt{74.5}\ \text{m/s} = 8.6\ \text{m/s}$$

Power

Power is the rate at which work is done by a force. Thus,

$$P = \frac{W}{t}$$

$$\text{power} = \frac{\text{work}}{\text{time}}$$

The more power something has, the more work it can perform in a given time. The SI unit for power is the **watt**, where 1 watt (W) = 1 joule/second (J/s).

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When a constant force **F** does work on a body that is moving at constant velocity ***v***, and if **F** is parallel to ***v***, the power involved is

$$P = \frac{W}{t} = \frac{Fd}{t} = F\left(\frac{d}{t}\right) = Fv$$

because $d/t = v$, that is,

$$P = Fv$$

$$\text{power} = (\text{force})(\text{velocity})$$

EXAMPLE: As part of an exercise activity, a 60-kg student climbs up a 7.5-m rope in 11.6 s. What is the power output of the student?

SOLUTION: The problem can be solved with the following equation:

$$P = \frac{W}{t} = \frac{Fd}{t} = \frac{mgh}{t} = \frac{(60\text{ kg})\left(9.8\text{ m/s}^2\right)(7.5\text{ m})}{11.6\text{ s}} = 380\text{ W}$$